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Suppression of interference in e–e scattering by the field of a strong electromagnetic wave

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Abstract. Electron–electron scattering in the presence of a strong electromagnetic field is investigated theoretically. The partial cross sections of multiphoton processes obtained are summed over the number of absorbed or emitted photons. The total cross section is shown to depend on the quantum nonlinearity parameter $\gamma = evE_0/\hbar\omega^2$. When $\gamma \gg 1$ the electromagnetic field suppressses interference of the direct and exchange channels of e–e scattering.

Electron–electron scattering and its resonance features in the presence of an intense plane electromagnetic wave (EMW) have been investigated by Oleinik (1967), Bös *et al* (1979), Bergou *et al* (1981), Kazakov and Roshchupkin (1983). Here we shall dwell upon some non-resonant features of this process. In other words we shall consider the regions not too close to the poles of the amplitudes of scattering.

As it will be shown below there is a rather pronounced difference between e–e scattering and scattering of electrons by some external static field in the presence of EMW (multiphoton stimulated bremsstrahlung, MSB). In the processes of MSB the main parameter of nonlinearity of partial multiphoton cross sections is equal to $\gamma = evE_0/\hbar\omega^2$ where E_0 and ω are the EMW field strength amplitude and frequency, v is the field-free electron velocity (Bunkin and Fedorov 1965, Bunkin *et al* 1972, Kroll and Watson 1973, Karapetyan and Fedorov 1978). The parameter γ contains the Planck constant $\hbar$ in the denominator and hence it has an essentially quantum mechanical origin. However, when one sums all the partial cross sections of MSB to calculate the absorption coefficient of EMW α all the essentially quantum mechanical contributions cancel each other; the nonlinearity parameter of α does not depend on $\hbar$ and is equal to v_E/v, where $v_E = eE_0/m\omega$ is the velocity amplitude of electron oscillations in EMW (Bunkin *et al* 1972, Karapetyan and Fedorov 1978).

In contrast to MSB, in the case of e–e scattering in the presence of EMW, these essentially quantum mechanical parts of partial multiphoton cross sections do not cancel each other completely after summation over the number of absorbed or emitted photons. As a result the main nonlinearity parameter of the summed cross section coincides with γ. When $\gamma \sim 1$ the electromagnetic wave affects essentially interference of the direct and exchange channels of e–e scattering.

Let the four-vector potential of EMW be given by

$$A(x) = a_1(e_1 \cos kx + \delta e_2 \sin kx) \qquad (1)$$

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where $a_1 = cE_0/\omega$, δ is the ellipticity parameter, $kx = \omega t - \boldsymbol{kx}$, $k = \omega n = \omega(1, \boldsymbol{n})$, is the photon four-momentum, $e_{1,2}$ are the unit polarisation four-vectors

$$e_j = (0, \boldsymbol{e}_j), \qquad e_j^2 = -1, \qquad e_j n = 0, \qquad j = 1, 2; \qquad e_1 e_2 = 0.$$

The process of e–e scattering will be studied here in the region of parameters

$$\eta = eE_0/m\omega c \ll 1, \qquad \hbar\omega \ll mc^2, \qquad \gamma \gg 1. \tag{2}$$

In the first Born approximation the process of e–e scattering in the field $A(x)$ (I) is determined by two diagrams described in figure 1 where the electron wavefunctions

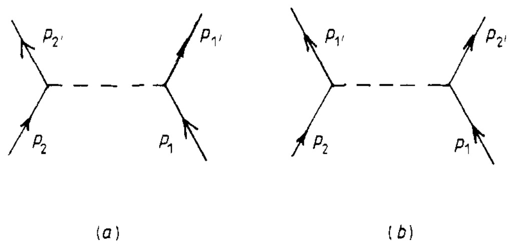

Figure 1. Diagrams of the direct (*a*) and exchange (*b*) channels of e–e scattering. In the presence of EMW the full lines correspond to the Volkov electron wavefunction. The broken lines correspond to the free virtual photon Green function.

before and after scattering are the well known Volkov functions (see e.g. Berestetsky *et al* 1980). The Fourier expansion of all the periodical functions entering in the S-matrix can be used to present the amplitude of scattering as a sum of partial amplitudes $S^{(l)}$ corresponding to absorption (if $l < 0$) or stimulated emission (if $l > 0$) of $|l|$ photons from or to the wave $A(x)$.

$$S = \sum_{l=-\infty}^{+\infty} S^{(l)} \tag{3}$$

where

$$S^{(l)} = \frac{\mathrm{i}(2\pi)^5 \mathrm{e}^2}{2(\varepsilon_1 \varepsilon_{1'} \varepsilon_2 \varepsilon_{2'})^{1/2}} \mathrm{e}^{\mathrm{i}\varphi_0} M^{(l)} \delta^{(4)}(p_{1'} + p_{2'} - p_1 - p_2 + l_* k) \tag{4}$$

$$M^{(l)} = \sum_{r=-\infty}^{+\infty} \left\{ b_1 b_2 \frac{(\bar{u}_{1'} \gamma^\mu u_1)(\bar{u}_{2'} \gamma_\mu u_2)}{[p_{1'} - p_1 + (l - r_*)k]^2} - [(1', 2') \to (2', 1')] \right\} \tag{5}$$

$$b_1 = \mathrm{e}^{-\mathrm{i}(l-r)\delta_1} \sum_{s=-\infty}^{+\infty} \mathrm{e}^{2\mathrm{i}s\delta_1} J_{l-r-2s}(\alpha_1) J_s(\tfrac{1}{2}\beta_1^-) \tag{6}$$

Here φ_0 is some constant phase, γ^μ $(\mu = 0, 1, 2, 3)$ are the Dirac matrices, $u_j (j = 1, 1', 2, 2')$ are the Dirac bispinors, $p_j = (\varepsilon_j, \boldsymbol{p}_j)$ $(j = 1, 2$ and $j = 1', 2')$ are the in- and out-going electron four-momentums.

$$l_* = l + \beta_1^+ + \beta_2^+, \qquad r_* = r - \beta_1^+ \tag{7}$$

$$\beta_j^\pm = \beta^\pm(p_{j'}) - \beta^\pm(p_j), \qquad j = 1, 2 \tag{8}$$

$$\beta^\pm(p) = \tfrac{1}{4}(1 \pm \delta^2)\eta^2 m^2/(kp) \tag{9}$$

b_2 is equal to b_1 (6) in which the index 1 is replaced everywhere by 2 and $l-r$ is replaced by r.

$$\begin{aligned} &\delta_j = \tan^{-1}(\delta \tan \varphi_j), \qquad \varphi_j = \measuredangle(\boldsymbol{e}_1, \boldsymbol{d}_{j\perp}) \\ &\alpha_j = \xi |\boldsymbol{e}_1 \boldsymbol{d}_j| \lambda_j, \qquad \lambda_j = (1+\delta^2 \tan^2 \varphi_j)^{1/2}, \qquad \xi = e a_1/\omega \end{aligned} \tag{10}$$

$$\boldsymbol{d}_j = \boldsymbol{p}_{j'}/(n p_{j'}) - \boldsymbol{p}_j/(n p_j), \qquad j = 1, 2 \tag{11}$$

$\mathbf{d}_{j\perp}$ is the projection of the vector $\boldsymbol{d}_j$ (11) upon the polarisation plane $(\boldsymbol{e}_1, \boldsymbol{e}_2)$.

In the centre of mass frame (CMF)

$$p_{1,2} = (\varepsilon_{\rm i}, \pm \boldsymbol{p}_{\rm i}), \qquad \boldsymbol{p}_{1',2'} = \tfrac{1}{2}\boldsymbol{P}_{\rm f} \pm \boldsymbol{p}_{\rm f} \tag{12}$$

where ε_j and $\boldsymbol{p}_j$ are the initial ($j = \mathrm{i}$) and final ($j = \mathrm{f}$) relative energy and momentum of the electrons, $\boldsymbol{P}_{\rm f}$ is the final momentum of CMF as a whole. Let θ be the CMF angle of scattering. Let for not too small θ and $|\theta - \pi|$

$$\theta \gg \omega/p_i, \qquad |\theta - \pi| \gg \omega/p_i, \qquad p_i = |\boldsymbol{p}_i| \tag{13}$$

the field strength E be restricted by the conditions

$$\eta^2 \ll \begin{cases} \omega/p_{\rm i}, & \text{if } v_{\rm i} \ll c \\ \omega\varepsilon_{\rm i}/m^2, & \text{if } \varepsilon_{\rm i} \gg mc^2. \end{cases} \tag{14}$$

These restrictions may be stronger than that given by inequality (2). Under the conditions (14) for any θ, $|\beta^{\pm}_{1,2}| \ll 1$ and hence $l_* \approx l$, $r_* \approx r$ and

$$b_1 \approx e^{-i(l-r)\delta_1} J_{l-r}(\alpha_1), \qquad b_2 \approx e^{-ir\delta_2} J_r(\alpha_2). \tag{15}$$

Also the $\delta^{(4)}$-function in equation (4) under the conditions (14) is reduced to

$$\delta^{(4)}(\ldots) = \tfrac{1}{2}\delta(\varepsilon_{\rm f} - \varepsilon_{\rm i} + \tfrac{1}{2} l\omega)\delta^{(3)}(\boldsymbol{P}_{\rm f} + l\boldsymbol{k}). \tag{16}$$

Now the squared four-momentums of virtual photons in equation (5) are equal to

$$\begin{aligned} q_1^2 &= [r - \tfrac{1}{2}l(1 - \boldsymbol{n}\boldsymbol{v}_{\rm f})]^2\omega^2 - [p_{\rm f} - p_{\rm i} - (r - \tfrac{1}{2}l)k]^2, \\ q_2^2 &= [r - \tfrac{1}{2}l(1 + n v_{\rm f})]\omega^2 - [p_{\rm f} + p_{\rm i} + (r - \tfrac{1}{2}l)k]^2 \end{aligned} \tag{17}$$

for the first and second diagrams in figure 1 respectively.

The amplitudes $S^{(1)}(4)$ have resonances when intermediate virtual photons in figure 1 become real, i.e. when

$$q_j^2 = 0, \qquad j = 1, 2 \tag{18}$$

(Oleinik 1967, Bös *et al* 1979).

Kazakov and Roshchupkin (1983) have shown that these resonances can occur only in the cases of forward and back scattering in the CMF i.e. if $\theta \sim \omega/p_i \ll 1$ or $|\theta - \pi| \sim \omega/p_i \ll 1$ for the figures 1(a) and (b) respectively. Hence the conditions (13) determine the non-resonance region where the amplitude $S^{(1)}(4)$ have no poles. In the non-relativistic limit the non-resonance regions for $S^{(1)}$ with $l \neq 0$ include both large and small θ and $|\theta - \pi|$ because the non-relativistic amplitude $S^{(1)}$ with $l \neq 0$ has no poles and no resonances at all (Bergou *et al* 1981, Kazakov and Roshchupkin 1983).

In the non-resonance region, equations (17) for q_j^2 are very much simplified because one can ignore there all the terms proportional to ω and ω^2. Under these conditions equation (5) for the matrix element $M^{(l)}$ is reduced to the form

$$M^{(l)}=\left(\mathrm{e}^{-\mathrm{i}\delta_1 l}C_l\frac{(\bar{u}_{1'},\gamma^{\mu}u_1)(\bar{u}_{2'}\gamma_{\mu}u_2)}{(p_{1'}-p_1)^2}-[(1',2')\rightarrow(2',1')]\right) \tag{19}$$

where

$$C_l=\sum_{r=-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}r(\delta_1-\delta_2)}J_{l-r}(\alpha_1)J_r(\alpha_2)=\mathrm{e}^{\mathrm{i}lt_1}J_l(x_1) \tag{20}$$

$$\mathrm{e}^{\mathrm{i}t_1}=(1/x_1)[\alpha_1+\alpha_2\,\mathrm{e}^{\mathrm{i}(\delta_1-\delta_2)}],\qquad x_1=[\alpha_1^2+\alpha_2^2+2\alpha_1\alpha_2\cos(\delta_1-\delta_2)]. \tag{21}$$

The differential partial cross sections $\mathrm{d}\sigma^{(l)}/\mathrm{d}\Omega$ are derived now in a routine way (Berestetzky *et al* 1980) after summation and averaging over polarisation of scattered and incident electrons. The result can be written in the form

$$\begin{aligned}\frac{\mathrm{d}\sigma^{(l)}}{\mathrm{d}\Omega}=\frac{e^4(\varepsilon^2+p^2)^2}{2\varepsilon^2}\rho_l\bigg[&\frac{F_1}{(\boldsymbol{p}_\mathrm{f}-\boldsymbol{p}_\mathrm{i})^4}J_l^2(x_1)+\frac{F_2}{(\boldsymbol{p}_\mathrm{f}+\boldsymbol{p}_\mathrm{i})^4}J_l^2(x_2)\\&+\frac{2\cos l\psi_0}{(\boldsymbol{p}_\mathrm{f}-\boldsymbol{p}_\mathrm{i})^2(\boldsymbol{p}_\mathrm{f}+\boldsymbol{p}_\mathrm{i})^2}\frac{3p^2-\varepsilon^2}{\varepsilon^2+p^2}J_l(x_1)J_l(x_2)\bigg]\end{aligned} \tag{22}$$

$$F_{1,2}=1+[(\varepsilon^2\pm p^2\rho_l\cos\theta)^2-2m^2p^2(1\mp\rho_l\cos\theta)]/(\varepsilon^2+p^2)^2 \tag{23}$$

$$\theta=\measuredangle(\boldsymbol{p}_\mathrm{i},\boldsymbol{p}_\mathrm{f}),\qquad \rho_l=\frac{|\boldsymbol{p}_\mathrm{f}|}{|\boldsymbol{p}_\mathrm{i}|}=\left[1-\frac{l\omega}{vp}+\left(\frac{l\omega}{vp}\right)^2\right]^{1/2}$$

where $\varepsilon=\varepsilon_\mathrm{i}$, $p=|\boldsymbol{p}_\mathrm{i}|$, $\psi_0=\delta_3-\delta_1+t_1-t_2$; t_2 and x_2 are equal to t_1 and x_1 in which δ_1, δ_2, α_1 and α_2 are replaced by δ_3, δ_4, α_3 and α_4; δ_j and α_j ($j=1,2,3,4$) are given by equations (10) with

$$\begin{aligned}\boldsymbol{d}_{1,2}&=\boldsymbol{v}_\mathrm{f}/(1\mp\boldsymbol{n}\boldsymbol{v}_\mathrm{f})-\boldsymbol{v}_\mathrm{i}/(1\mp\boldsymbol{n}\boldsymbol{v}_\mathrm{i})\\ \boldsymbol{d}_{3,4}&=\boldsymbol{v}_\mathrm{f}/(1\pm\boldsymbol{n}\boldsymbol{v}_\mathrm{f})+\boldsymbol{v}_\mathrm{i}/(1\mp\boldsymbol{n}\boldsymbol{v}_\mathrm{i}).\end{aligned} \tag{24}$$

Let us consider at first the case of relativistic energies of electrons in the CMF. Then $|l|\omega/vp\leqslant 4m/p\ll 1$ and hence $\rho_l=1$ (23). Under this assumption the partial cross sections $\mathrm{d}\sigma^{(l)}/\mathrm{d}\Omega$ are easily summed over l to give

$$\begin{aligned}\mathrm{d}\sigma/\mathrm{d}\Omega&=\sum_l\mathrm{d}\sigma^{(l)}/\mathrm{d}\Omega\\&=\frac{e^4(\varepsilon^2+p^2)^2}{4\varepsilon^2p^4}\left(\frac{F_1}{8\sin^4\frac{1}{2}\theta}+\frac{F_2}{8\cos^4\frac{1}{2}\theta}+\frac{3p^2-\varepsilon^2}{p^2+\varepsilon^2}\frac{J_0(x)}{\sin^2\theta}\right)\end{aligned} \tag{25}$$

where

$$x=(x_1^2+x_2^2-2x_1x_2\cos\psi_0)^{1/2}. \tag{26}$$

When $E_0\rightarrow 0$, $x\rightarrow 0$, $J_0(x)\rightarrow 1$ and equation (25) turns into the field-free Möller cross section of e–e scattering $\mathrm{d}\sigma_{\mathrm{M\ddot{o}l}}/\mathrm{d}\Omega$. In this limit the first, second and third terms in the brackets of equation (25), correspond to the direct and exchange channels of scattering and to their interference. When $E_0\neq 0$, $|J_0(x)|<1$ and hence the effect of interference in e–e scattering is lowered due to the influence of EMW. A degree of influence of EMW upon the cross section of scattering may be characterised by the parameter y determined by the equations

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=(1-y)\frac{\mathrm{d}\sigma_{\mathrm{M\ddot{o}l}}}{\mathrm{d}\Omega} \tag{27}$$

or explicitly

$$y=\left(\frac{[4-(m/p)^4]\sin^2\theta}{(3+\cos^2\theta)^2-(1+3\cos^2\theta)[4-(1+\varepsilon^2/p^2)^2]}\right)(1-J_0(x)). \qquad (28)$$

Expression (26) for the argument x of the Bessel function in equations (25) and (28) is simplified in the case of a linearly polarised EMW $\delta=0$:

$$x_{\text{lin}}=|\alpha_1+\alpha_2-\alpha_3-\alpha_4|$$

where α_j ($j=1,2,3,4$) are given by equations (10), (24).

In the ultrarelativistic limit $1-v\ll 1$ equations (27), (28) yield

$$\frac{d\sigma_{\text{ur}}}{d\Omega}=\left(\frac{d\sigma_{\text{Möl}}}{d\Omega}\right)_{\text{ur}}\left(1-\frac{4\sin^2\theta}{(3+\cos^2\theta)^2}(1-J_0(x))\right). \qquad (29)$$

The conditions of applicability of equations (25), (28) are given by inequalities (2), (13) and $\varepsilon\gg\eta^2m^2/\omega$.

The argument of the Bessel functions (26) in equations (25), (28), (29) is of the order of quantum parameter γ calculated at $v\approx c$. When $\gamma\ll 1(\eta\ll\omega/m)$ the EMW (1) only very weakly affects the process of e–e scattering (in this case $y\sim\gamma^2\ll 1$ in accordance with the first-order perturbation theory term). When $\gamma\gtrsim 1(\eta\gtrsim\omega/m)$ the influence of EMW on the process of e–e scattering becomes more essential because in this case $1-J_0(x)\sim J_0(x)\sim 1$ in equations (25), (28), (29). If at last $\gamma\gg 1$ the field of EMW almost completely suppresses interference of the direct and exchange channels in e–e scattering, the corresponding term in equation (25) being almost completely excluded: $J_0(x)\approx 0$. The conditions $\gamma\gg 1$ and $\eta\ll 1$ for $\omega\approx 3\times 10^{15}\ \text{s}^{-1}$ are satisfied if the electric field strength of the wave E_0 is within the range

$$10^4\text{–}10^5\ \text{V/cm}\ll E_0\ll 10^{10}\text{–}10^{11}\ \text{V/cm}. \qquad (30)$$

Under these conditions for example in the ultrarelativistic limit for $\theta\doteq\frac{1}{2}\pi$ equation (29) gives

$$d\sigma_{\text{ur}}/d\Omega=\tfrac{5}{9}(d\sigma_{\text{Möl}}/d\Omega)_{\text{ur}} \qquad (31)$$

i.e. under the influence of EMW the cross section $d\sigma_{\text{ur}}/d\Omega$ becomes almost twice less than the corresponding field-free cross section.

Let us consider now the non-relativistic region of electron energies in the EMF $v\ll 1$, supposing also that the frequency of the field ω is not too large

$$\omega\ll p^2/m \qquad (32)$$

Now $\delta_1=\delta_3$ and $\delta_2=\delta_4$ in equations (10), (21) and the partial multiphoton cross sections (22) are given by

$$\frac{d\sigma^{(l)}}{d\Omega}=\left(\frac{4e^2}{mv^2}\right)^2\rho_l\left(\frac{J_l^2(2\alpha_1)}{\gamma_1^4}+\frac{J_l^2(2\alpha_3)}{\gamma_2^4}-\frac{\cos l(\delta_1-\delta_3)}{\gamma_1^2\gamma_2^2}J_l(2\alpha_1)J_l(2\alpha_3)\right) \qquad (33)$$

where

$$\alpha_{1,3}=\gamma|\boldsymbol{e}_1\boldsymbol{\gamma}_{1,3}|\lambda_{1,3}, \qquad \boldsymbol{\gamma}_{1,3}=\rho_k\boldsymbol{n}_f\mp\boldsymbol{n}_i$$
$$\boldsymbol{n}_j=\boldsymbol{v}_j/|\boldsymbol{v}_j|(j=\text{i},\text{f}), \qquad \rho_l=[1-4l\omega/mv^2]^{1/2}. \qquad (34)$$

These equations are applicable under the conditions (13) plus the condition of the first Born approximation $v \gg \alpha = \frac{1}{137}$ plus the restriction $\eta^2 \ll \omega/mv$. The latter restriction together with the assumption (32) restrict the parameter η by the inequality $\eta \ll v^{1/2}$ which is stronger than the condition (2) ($\eta \ll 1$). Under assumption that the field parameter is even stronger restricted $\eta \ll v$ the difference $\rho_l - 1 \sim |l|\omega/mv^2 \lesssim \eta/v$ becomes small. In this case again the partial cross sections $d\sigma^{(l)}/d\Omega$ (33) can be summed to give

$$\begin{aligned} d\sigma_{nr}/d\Omega &= \sum_l d\sigma_{nr}^{(l)}/d\Omega \\ &= \left(\frac{d\sigma_{\text{Möl}}}{d\Omega}\right)_{nr} \left(1 + \frac{\sin^2\theta}{1+3\cos^2\theta}[1 - J_0(x)]\right) \end{aligned} \tag{35}$$

where now

$$x = 2|\alpha_1 - \alpha_3|, \qquad \alpha_{1,3} = \gamma|\boldsymbol{e}_1(\boldsymbol{n}_f \mp \boldsymbol{n}_i)|\lambda_{1,3}. \tag{36}$$

This result can be derived also directly from equations (27), (28) in the non-relativistic limit $v \to 0$. The conditions of applicability of equation (35) are given by inequalities (13) plus

$$\eta^2 \ll \omega/mv, \qquad \eta \ll v. \tag{37}$$

Equation (35) shows that again the field of EMW diminishes the effect of interference in e–e scattering in the non-relativistic limit when $\gamma \gtrsim 1$. For example, when $\gamma \gg 1$, $\theta = \frac{1}{2}\pi$ equation (35) gives

$$d\sigma_{nr}/d\Omega = 2(d\sigma^{\text{Möl}}/d\Omega)_{nr} \tag{38}$$

i.e. asymptotically for large γ and $\theta = \frac{1}{2}\pi$ the non-relativistic cross section in the field of EMW is twice as large as the field-free Möller cross section. A similar result has been derived numerically in the case of a very strong field $\eta = 1$ by Bös *et al* (1979).

The condition $\gamma \gg 1$ is satisfied together with restrictions (37) e.g. for $v \sim 10^{-2}c$, $\omega \sim 3 \times 10^{+15}$ s^{-1}, $E_0 \sim 10^7$–10^8 V/cm. This field is only moderately strong and is obtainable with the aid of modern lasers.

As a résumé we would like to repeat and to emphasise our main qualitative conclusions. The cross section of e–e scattering in the field of EMW summed over the number of absorbed or emitted photons depends on the quantum nonlinearity parameter $\gamma \sim evE_0/\hbar\omega^2$. When $\gamma \gtrsim 1$ the field induced effect is a suppression of interference of the direct and exchange channels in e–e scattering. Due to the quantum mechanical nature of the nonlinearity parameter γ this field induced effect can take place when the field strength of EMW is not too strong and in particular when the usual quantum-electrodynamical field parameter η is very small. All these features of e–e scattering make this process different from MSB in the case of electron scattering in an external field of ions, atoms, etc. In MSB, summation over the number of absorbed or emitted photons excludes all the essentially quantum mechanical peculiarities of scattering. The reason why MSB is so very different from e–e scattering is in the exchange between two electrons scattered by each other. This effect has no analogy in MSB. Exchange is an essentially quantum mechanical feature of the two-electron system. Probably this is the reason for which the cross section of e–e scattering summed over the number of absorbed or emitted photons conserves some essentially quantum mechanical features and, in particular, depends on the quantum mechanical nonlinearity parameter γ.

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